1.

a. A filamentary loop in the shape of an n-gon, that is to say, a regular polygon of n sides. Show that the centre of the polygon has

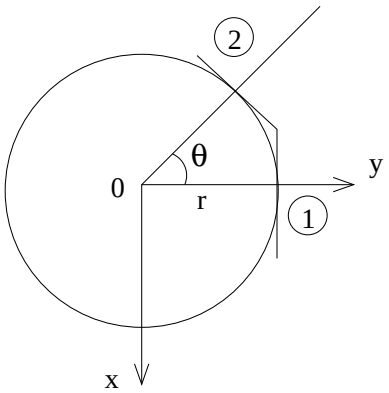
$$H = \frac{nI}{2\pi r}\sin\frac{\pi}{n}$$

where r is the radius of the circle circumscribing the polygon.

b. Find $H(n)$ for $1 \le n \le 15$ and tabulate the results. Then plot the graph of H against n and give a brief discussion. (Assume, for example, $I = 1$A and $r = 1$m.) [20]

Solution. Let θ be the angle between the two rays perpendicular to adjacent edges of the n-gon. Then,

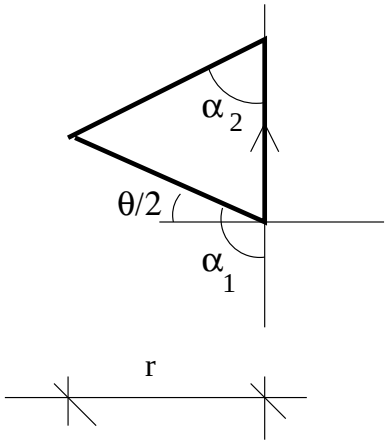
$$\theta = \frac{2\pi}{n}$$



Assume that

- the n-gon is on the plane $z = 0$
- the current flows as shown in the anticlockwise direction
- the ray perpendicular to edge number 1 lies on the y-axis

Next, look at a pie-section built on a single whole edge, say Edge 1.



We have $\rho = r$ and $\mathbf{a}_\phi = \mathbf{a}_z$.

$$\cos\alpha_2 = \cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \sin\frac{\theta}{2} = \sin\frac{\pi}{n}$$

$$\cos\alpha_1 = \cos\left(\frac{\pi}{2} + \frac{\theta}{2}\right) = -\sin\frac{\theta}{2} = -\sin\frac{\pi}{n}$$

$$\begin{aligned}\mathbf{H}_1 &= \frac{I}{4\pi r}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_z \\ &= \frac{I}{4\pi r}\left(2\sin\frac{\pi}{n}\right) = \frac{I}{2\pi r}\sin\frac{\pi}{n}\mathbf{a}_z\end{aligned}$$

An n-gon has n sides, hence

$$H = \frac{nI}{2\pi r}\sin\frac{\pi}{n}$$

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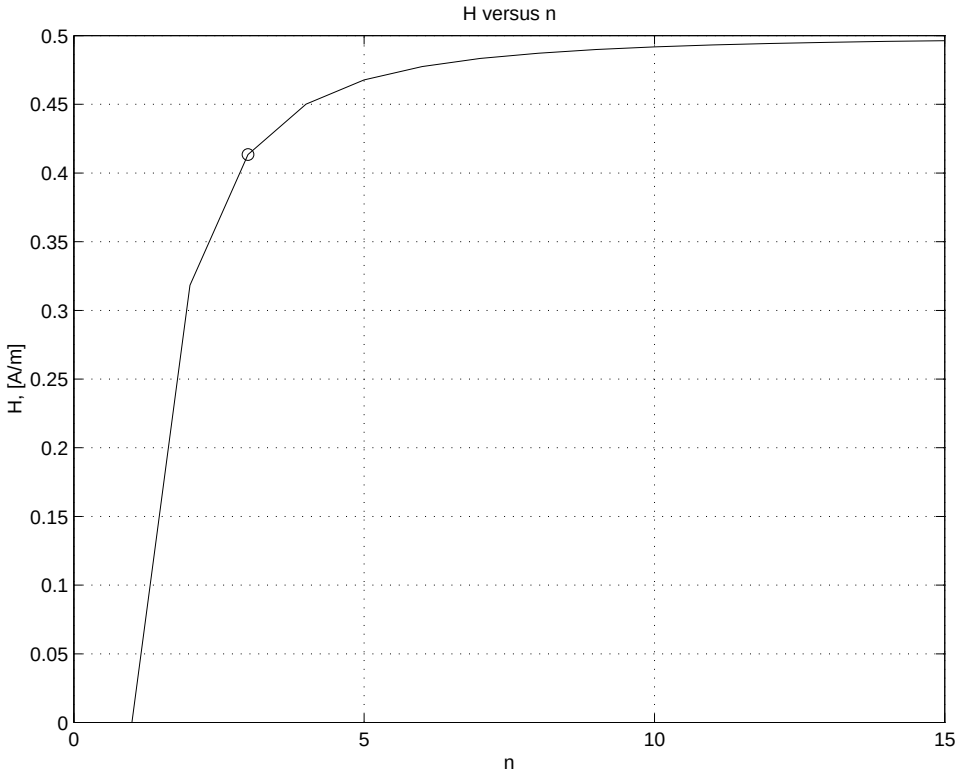
b. Let $r = 1\text{m}$ and $I = 1\text{A}$.

n	H
1	0.0000
2	0.3183
3	0.4135
4	0.4502
5	0.4677
6	0.4775
7	0.4834
8	0.4872
9	0.4899
10	0.4918
11	0.4932
12	0.4943
13	0.4951
14	0.4958
15	0.4964

The plot is the following.



For a physically meaningful n-gon $n \geq 3$. H increases from 0.4135 for an equilateral triangle, where $n = 3$, towards a steady-state value of 0.5, which corresponds with the value obtained from a circular ring

$$\frac{I}{2r} \lim_{n\to\infty} \left(\frac{n}{\pi} \sin \frac{\pi}{n}\right) = \frac{I}{2r}.$$

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